JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 90, NO. C6, PAGES 11,779–11,782, NOVEMBER 20, 1985

# Cautionary Remarks on the Spectral Interpretation of Turbulent Flows

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Spectral slopes are shown to be only very weak constraints for testing turbulence theories. They are primarily a consequence of applying spectral analysis to flows that are not wavelike but contain simple structures represented by a broad extension in wave number space.

## INTRODUCTION

Spectral analysis of turbulent flows and theories that attempt to explain observed spectra, in particular spectral slopes, have for some time now been a major technique in attempts to account for turbulent flow behavior. This note is a cautionary one, concerning what we feel is a much overstressed, although easy to measure, aspect of turbulent flows.

We will show that the shapes of spectra are only very weak constraints for testing theories and information concerning the structure of flows: for example the positions of fronts relative to large-scale coherent structures reside in phase information of individual realizations. This is just the information that is lost through ensemble averaging to obtain the power spectrum and is not dealt with in spectral theories of turbulence.

## AN EXAMPLE OF SPECTRAL ANALYSIS

We have chosen for analysis an infrared image of the ocean sea-surface temperature. The actual image used is shown in Figure 1*a*. Details concerning the associated field experiment can be found in *Flament et al.* [1985]. An image was chosen, since it provides us with a panoramic view that can be readily interpreted visually in a pattern recognition sense, as well as data suitable for spectral analysis. Data from a single probe, or even a sizable number of probes, cannot provide the necessary panoramic view needed to recognize the disparity in scale between the fronts and eddies visible in this image. The number of pixels comprising this image is 256 × 256, with a 1-km resolution. The center of the image is at 38°30'N, 125°00'W.

The original image (Figure 1*a*) was multiplied by a 25% half-cosine window to bring the edge of the frame to a constant value. The windowed image is shown in Figure 1*b*. A two-dimensional Fourier transform was made of this image, the phase and magnitude of which are shown in Plate 1 (*a*, *b*). (Plate 1 can be found in the separate color section in this issue.) The azimuthal average of the transform magnitude is shown as a spectral density in Figure 2. The spectral slope of $k^{-4}$ differs from the predictions of the spectral slope of temperature from simple quasi-geostrophic turbulence theories. Rather than discussing this discrepancy, we will instead show that the spectral slope is only a very weak constraint on the dynamics of the flow.

We are well aware that images of the type shown here are not ensemble averages but single realizations of a complex turbulent flow. For turbulent flows, ensemble averages are usually expressed either in terms of autocorrelation coefficients or the directly related power spectral density; neither retains information about the phase of the Fourier transform of each individual realization. This is also the case with spectral theories of turbulence that deal with power spectra. We might ask what other types of realizations, other than the original turbulent flow, would have an identical spectral slope.

Therefore we generated a random phase and used the original magnitude (Plate 1*b*) to compute the inverse Fourier transform. The random phase used and the resultant image are shown in Plates 2 and 3. (Plates 2 and 3 can be found in the separate color section in this issue.) Obviously, there is no resemblance between the cloudlike structures of this image and the original temperature field (Figure 1). An ensemble of such cloudlike structures yields the same average energy spectrum as the jet and associated eddies; a spectral theory of turbulence based on a closure for the power spectrum [cf. *Charney*, 1971] applies equally well to these cloudlike structures as to the original temperature field of the geophysical flow.

Now we present three images for which we retain the phase information of Plate 1*b* but place less emphasis on the observed spectrum. Figure 3*a* is the image produced by applying an inverse Fourier transform on the original phase information and a synthetic azimuthally constant spectral density of slope $k^{-4}$. Most of the structure of the original turbulent flow can be seen. To further stress the unimportance of the spectral slope, we now recompute the images of applying an inverse Fourier transform on the original phase and synthetic spectral densities with spectral slopes differing from the original $k^{-4}$ slope. This image processing technique is known as coefficient rooting [*Andrews et al.*, 1972]. Figure 3*b* shows the image computed with a synthetic spectral slope of $k^{-3}$, and Figure 3*c* shows a similar image with a synthetic spectral slope of $k^{-2}$.

## DISCUSSION

The effect of changing the spectral slope, yet retaining all the phase information, has been to emphasize that the distinctive signature of the coherent structures in the turbulent flow is the relative position of small scales with respect to large scales. This is particularly evident in Figure 3*c*, in which the $k^{-2}$ spectral density has the effect of greatly amplifying the magnitude of the small details, in effect a kind of high-pass filtering. This is, of course, not new. The line drawings of turbulent flows by Leonardo da Vinci, as seen for example in Figure 4, contain no magnitude information but only phase information. Nonetheless these drawings convey an amazing grasp of the essential flow structures. In a different context the importance of phase in signals of various kinds has recently been discussed by ***Oppenheim and Lim*** [1981].

The $k^{-4}$ spectral slope observed in Figure 2 in the wave number range of $10^{-2}$ to $5 \times 10^{-1}$ km$^{-1}$ is primarily a spec-



Paper number 5C0553.
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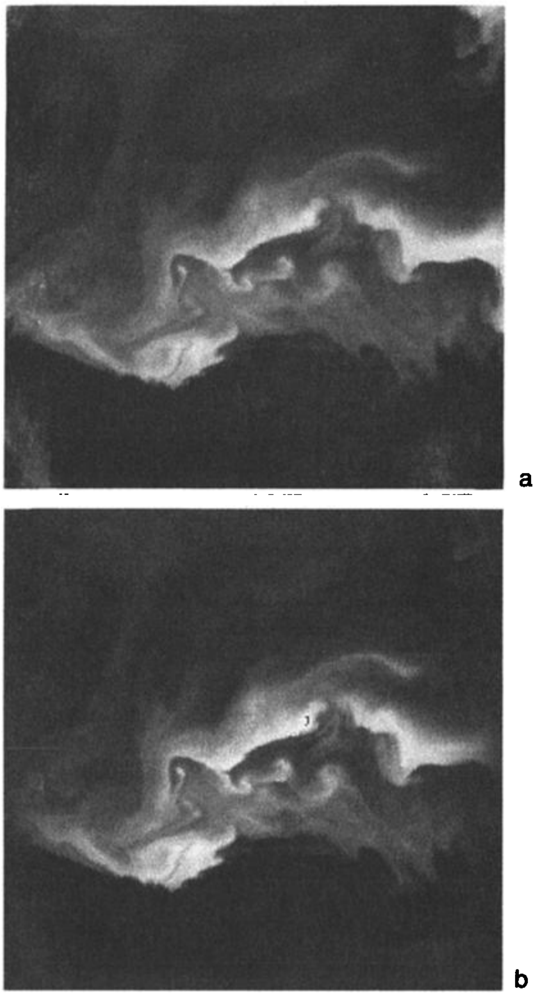

Fig. 1. (*a*) Original infrared image of the ocean sea-surface temperature of a filament and associated eddies off the northern coast of California [from *Flament et al.*, this issue]. The image is made of 256 × 256 pixels with 1-km resolution. (*b*) Identical to Figure 2*a* but windowed to bring the edge of the frame to a constant value.

tral manifestation of the existence of intersecting smooth gradients. Relative temperature along a north-south line through the center of the windowed image (Figure 1*b*) is shown in Figure 5. This temperature variation is dominated by two intersecting ramps, the transform of which is

$$F(k) = 4a^2 \frac{\sin^2 kl}{k^2}$$

The spectral density of this transform is shown in Figure 6; the side lobes fall off with a slope of $k^{-4}$. The azimuthally averaged spectral density of an image in which temperature variations are made up of smooth intersecting ramps of differing lengths will then have a $k^{-4}$ spectral slope, the side lobes being smeared out in the averaging. The tapering off of the spectral density at spatial frequencies higher than $5 \times 10^{-1}$ $\text{km}^{-1}$ is due to instrumental noise of the scanning radiometer.

## CONCLUSION

These cautionary remarks on the spectral interpretation of turbulent flows were prompted by some recent attempts to compare satellite-derived spectra with predictions of theories of geostrophic turbulence. *Deschamps et al.* [1981] performed a statistical analysis of the sea surface temperature field by means of the structure function giving a spatial equivalent of the spectral density with power law exponent varying from 1.5 to 2.3. There is also a long history of spectral analyses of sea surface temperature from ships and aircraft, including those of *Voorhis and Perkins* [1966], *McLeish* [1970], *Saunders* [1972], and *Holladay and O'Brien* [1975]. Similar spectral analysis of sea surface pigment has been made by *Gower et al.* [1980], and problems associated with its interpretation as a dynamically related quantity, such as temperature, as opposed to a passive tracer have been pointed out clearly by *Lesieur and Sadourny* [1981]. The temperature field analyzed in our image was in fact a mix of both a dynamically related variable and a purely passive one, since off the coast of Northern California, large *T-S* variations exist, and the surface temperature field is not consistently correlated with the surface density field.

The spectral slopes of the analyses above are usually compared with predictions of two-dimensional or quasi-geostrophic turbulence theories, well known examples of which are those of *Kraichnan* [1967], *Charney* [1971], and *Salmon* [1978]. These theories attempt to model turbulence in terms of interactions among different wave numbers, often so-called eddies, of the energy spectrum. However, the spectral slopes observed are a consequence of the application of spectral analysis to structures represented by a broad extension in wave number space, as shown for example in Figures 5 and 6. The panoramic view provided by the satellite infrared image (Figure 1*a*) does not contain a cascade of eddies or an inertial subrange, although the power spectrum slope alone might lead one to believe that such a subrange were present.

An essential characteristic of these two-dimensional turbulent flows is the relative position of small-scale eddies and sharp fronts with respect to larger scales such as the offshore flowing jet. *Flament et al.* [1985] argue that the scale of the sharp fronts (~300 m) represents a balance between diffusion resulting from small-scale (~30 m) turbulence in the surface mixed layer and large-scale (~30 km) horizontal strain apparent in a sequence of images similar to that shown in Figure 1. *Saffman* [1971] has also shown that the vorticity distribution in a field of random two-dimensional vorticity develops discontinuities, and he has calculated the energy spectrum that behaves like $k^{-4}$ for large values of the wave number. He points out, as does *Charney* [1971], that the experimental evidence is not capable of distinguishing conclusively between either a $k^{-3}$ or $k^{-4}$ dependence.

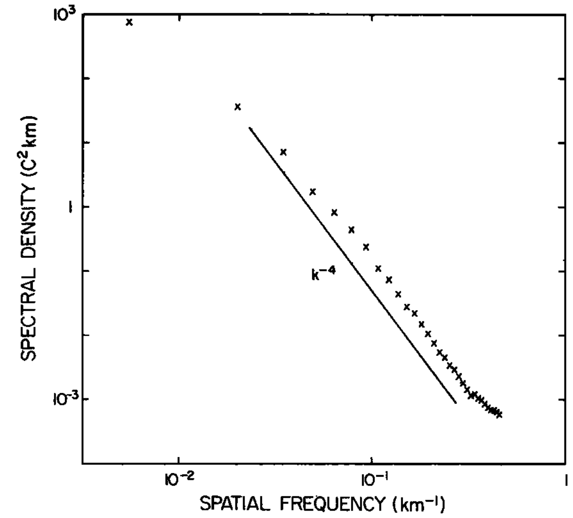


Fig. 2. Spectral density from an azimuthal average of the two-dimensional Fourier magnitude shown in Plate 1*a*.

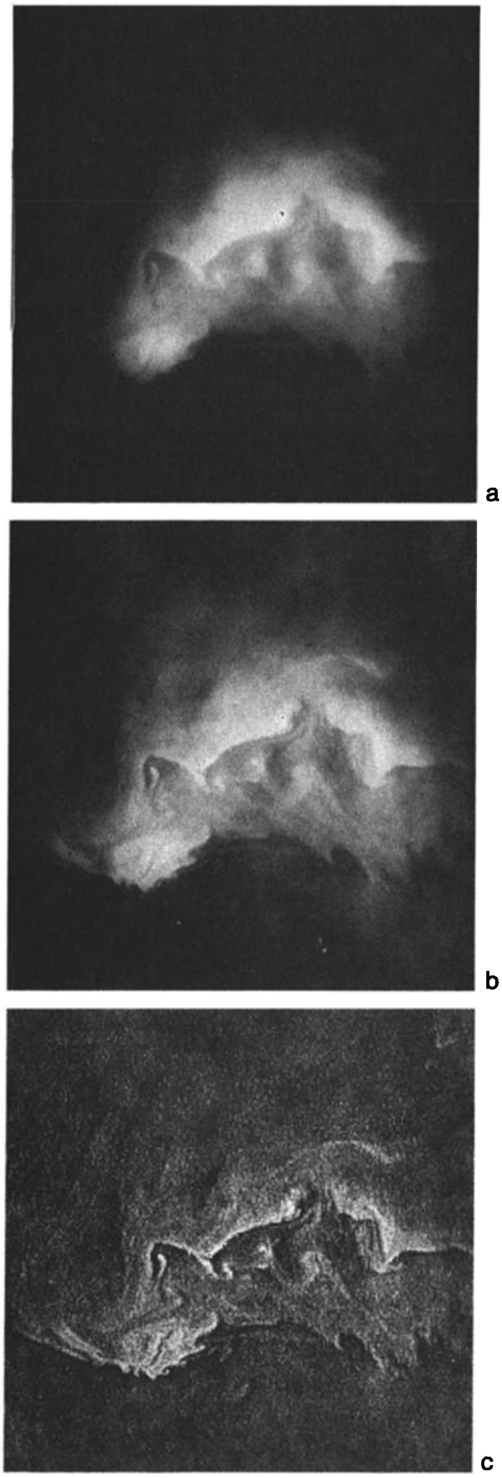


Fig. 3. Inverse two-dimensional Fourier transform using phases (shown in Plate 1*b*) of the original windowed image and synthetic Fourier amplitudes with spectral density slopes of (*a*) $k^{-4}$, (*b*) $k^{-3}$, and (*c*) $k^{-2}$.

An interesting numerical experiment of *Fornberg* [1977] for two-dimensional decaying turbulence illustrates the possibility of significant phase correlations between Fourier components. His results showed a $k^{-4}$ spectrum after a sufficiently long time. The phases of the Fourier components were then scrambled in a random manner, keeping the magnitude the same, as we did in Plates 2 and 3 here. Without the proper phase correlations, an "unnatural" burst of high-frequency components is generated, bringing the spectral slope from fourth to third power, which eventually relaxes back to fourth power.

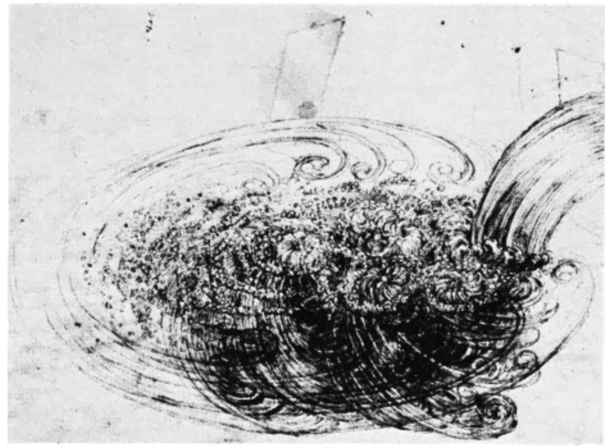

Fig. 4. Line drawing of a turbulent outfall by L. da Vinci (personal communication, 1508) containing no magnitude information.

Similar cautionary remarks apply to three-dimensional turbulence. Flow visualizations by *Brown and Roshko* [1974] of perhaps the simplest such flow imaginable, a developing free shear layer, do not exhibit the continuous cascade to smaller scales in the sense of the theory of *Kolmogoroff* [1941]. Recently, *Corcos and Sherman* [1984] have accounted for many aspects of this flow from a deterministic viewpoint. Nonetheless, this flow has a well-established fully developed turbulent $k^{-5/3}$ pedigree, as has been observed by *Champagne et al.* [1976]. Since the theory of *Kolmogorov* [1941] is based on physical assumptions fundamentally different from the observations, the spectral slope alone is inadequate to differentiate between theories.

From the perspective of a Fourier decomposition, the essential information regarding structure of the turbulent flow is contained in the two-dimensional phase. Unfortunately, phase is difficult to interpret (see Plates 1 and 2) and lost through averaging to get the power spectrum. Although higher-order spectra might well be computed (e.g., bispectra and energy

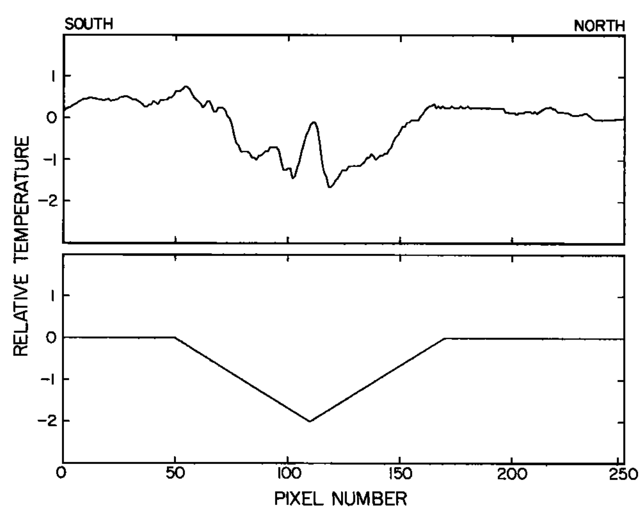


Fig. 5. (Top) Relative temperature on a north-south line through the center of the windowed image (Figure 1*b*). (Bottom) Idealized temperature on the same north-south line showing dominance of two intersecting ramps.

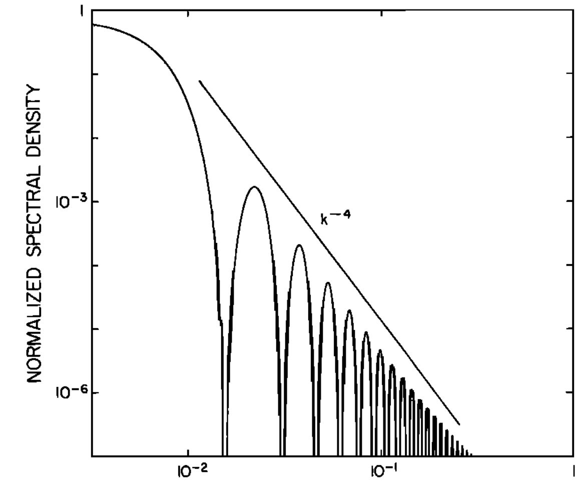


Fig. 6. Normalized spectral density of the idealized temperature distribution shown in Figure 5. Side lobes fall off with a slope of $k^{-4}$.

transfer spectra), and modern statistical theories are based around these higher-order spectra, it is not at all clear how the crucial phase information is obscured by these higher-order averages for coherent structures of varying size. Spectral analysis is a straightforward technique, but we are questioning the usefulness of its application to flows that are not wavelike but contain clearly recognizable signatures of coherent structures like jets, developing eddies and associated fronts as in the flow analyzed here. These coherent structures will always have a broad extension when represented in a Fourier wave number space. Equivalent statements can also be made for correlations and closure theories based around them, since correlations and power spectra are formally related to each other. We are concerned that very high order ensemble statistical quantities are needed to account for existence of fronts or intermitency in these flows. Turbulence closure theories at even higher order than those proposed now would be increasingly more obscure.

*Acknowledgments.* Henry Stommel commented on a preprint of the manuscript that "a power spectrum of some bars of Beethoven would be rather an oversimplification too; maybe just enough to decipher the key." Thanks to Stephen Pazan for telling us about analogous work in the field of image processing and to Geoffrey Vallis for helpful criticism. Our research is funded by the Office of Naval Research and the National Science Foundation. The image processing was carried out at the Scripps Satellite Oceanography Facility, which is partially supported by an NSF/ONR/NASA block grant.

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(Received May 16, 1985;
accepted July 8, 1985.)

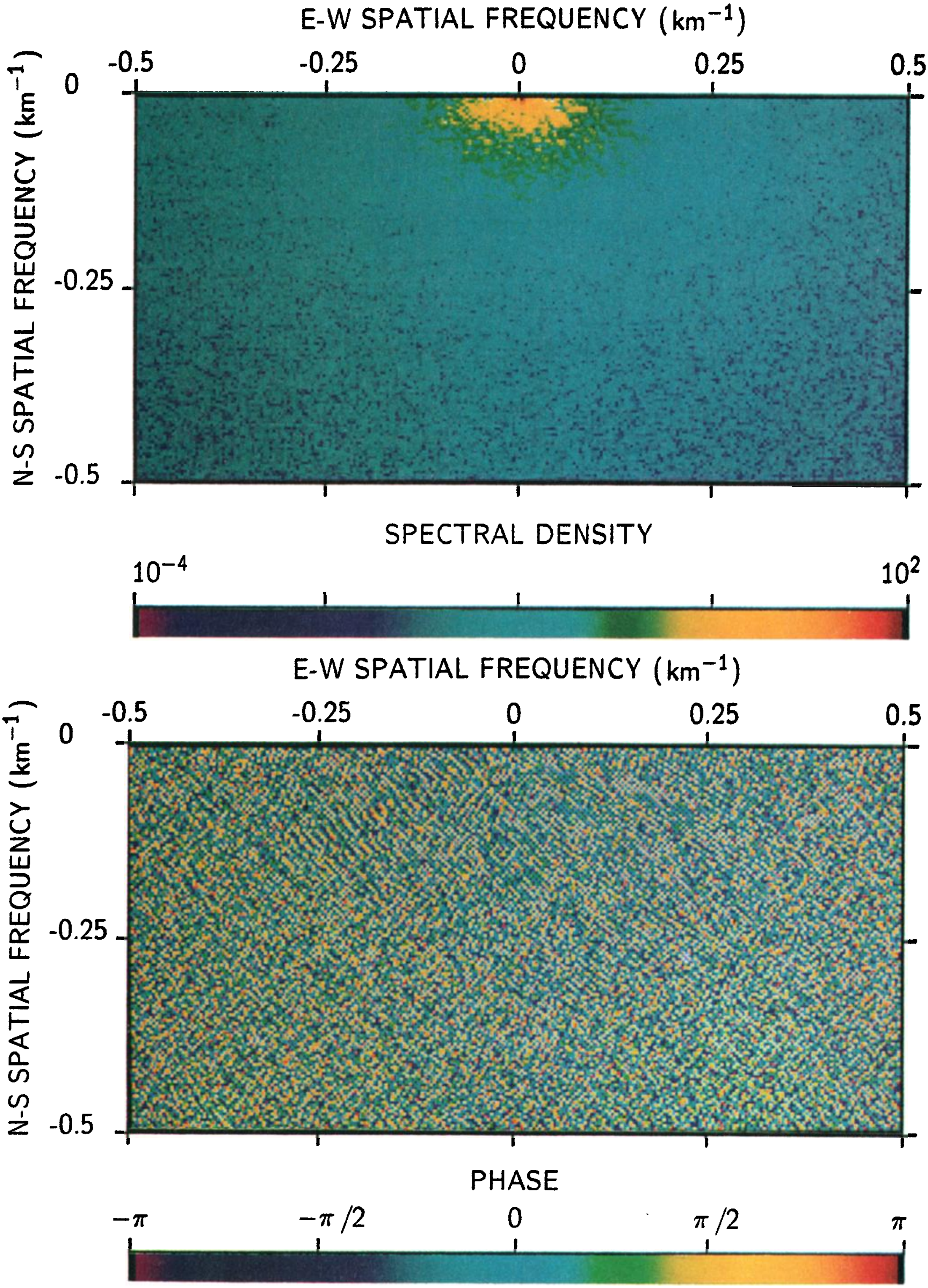


Plate 1 [*Armi and Flament*]. Two dimensional Fourier transform (*a*) magnitude and (*b*) phase. Only the lower half of the transform is shown because the upper half is the complex conjugate, since the original image is real.

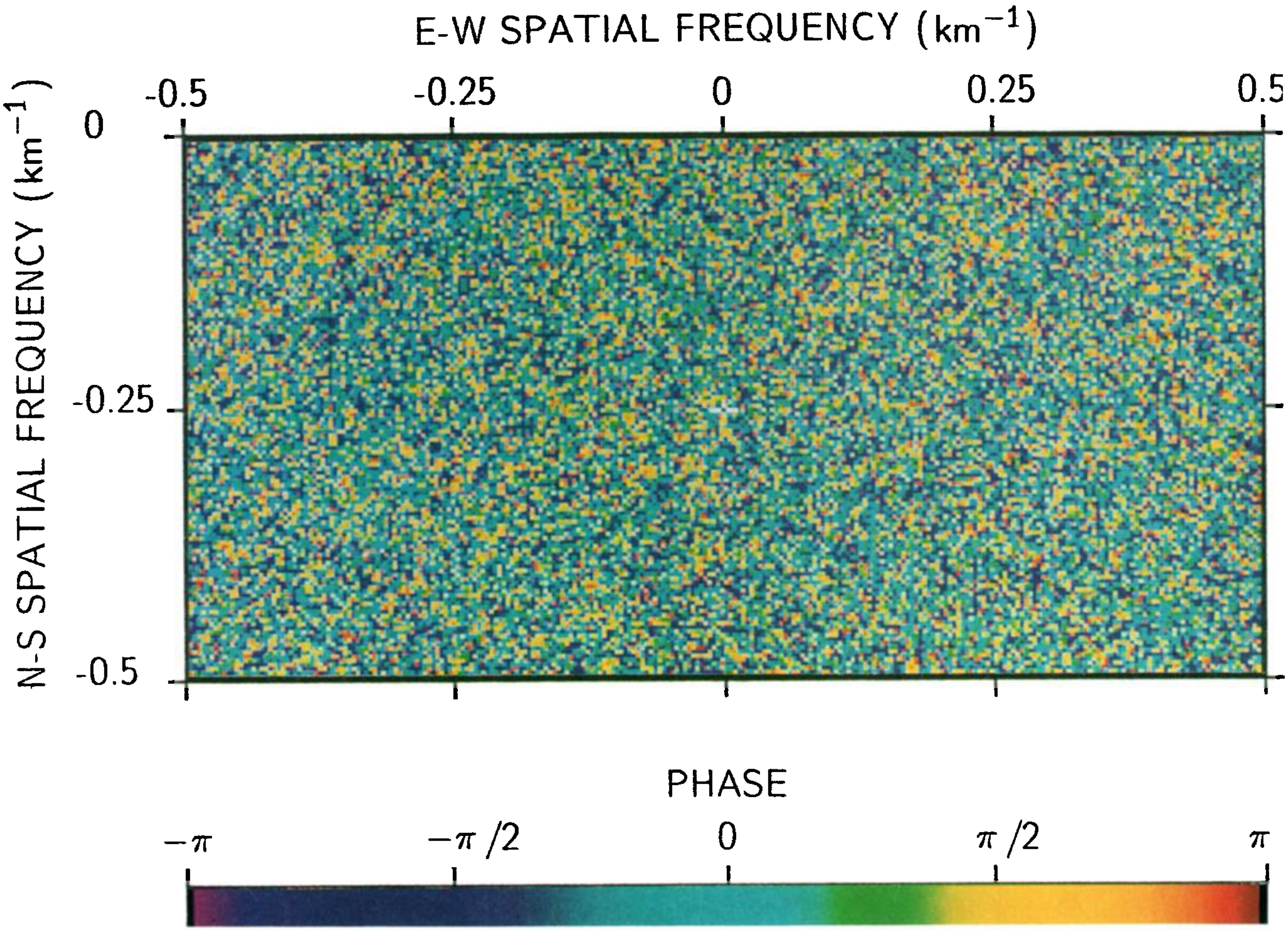


Plate 2 [*Armi and Flament*]. Random phase uniformly distributed from $-\pi$ to $+\pi$.

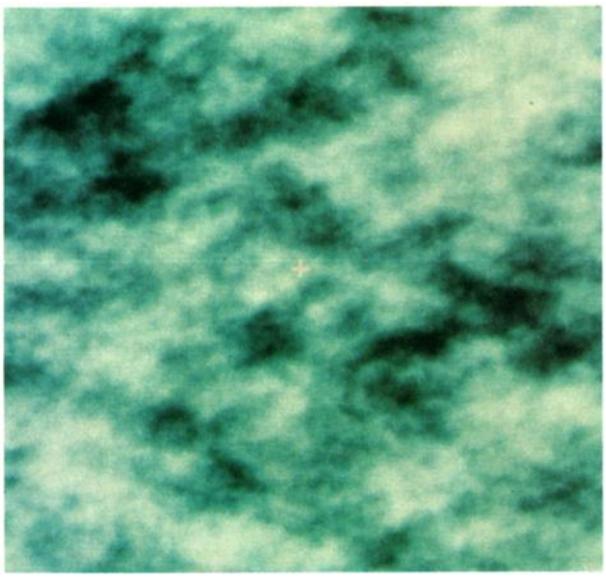

Plate 3 [*Armi and Flament*]. Inverse two-dimensional Fourier transform using the Fourier magnitude (Plate 1*a*) of the original windowed image and the random phase shown in Plate 2.